Numerical Analysis of Geometry Effect on Surface Charge and Electric Field in Current-Carrying Conductor

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This paper presents a numerical method to calculate surface charge and electric field distributions in current-carrying conductor. The proposed method is based on the finite element method, so that it can be applied to conductors of arbitrary geometry. In this analysis, two Laplace equations are sequentially solved because the material properties of the conductor and the region outside the conductor are incompatible in an equation. Accurate calculation of surface charge and electric field distributions using the proposed method can widen understanding of physical phenomenon due to the geometry effect. To show usefulness of this method, numerical models with different shaped conductor are analyzed.

***Index Terms*—Conductor, Electric field, Finite element method, Surface charge**

I. INTRODUCTION

Electrodes, which voltage is applied, generate electric field in the whole space. When a conductor is connected between the electrodes, the electric field is rearranged and current flows along to the conductor. The cause of the rearrangement was known as surface charge on the conductor decades ago [1]. The three roles of the surface charges are: to maintain the potential around the circuit, to provide the electric field in the outside of the conductor, and to assure the confined flow of current by generating an electric field that is parallel to the wire [2].

Accurate calculation of surface charge and electric field distributions contributes the efficient design of the facilities and also prevents severe accidents. For example, lightning protection system protects facility structures from sudden impact by lightning. The system consists of air termination, down conductor, earth termination, and surge arrester [3]. The bent parts of the down conductor are usually unavoidable due to the dependence on the structure shape. As the bending is sharp, the impedance of the system is increased, and high current can damage the facilities because of lighting.

Another example is the capacitance calculation of parasitic circuit parameter in transformer windings. Resonance due to this capacitance and the inductance of the transformer can cause unnecessary power loss [4]. The parasitic capacitance is also important issue in PCB highly integrated with components. Leakage field in the PCB affects nearby circuit components, so unexpected performance could occur [5].

Many prior studies analytically derived the surface charge distribution of simple geometries: straight wire, coaxial wire, circular loop, etc. [6]. The practical conductors, which are used in the laboratory and the industry, are not such a simple. To overcome the limitation, some studies used the numerical analysis. Since the material properties of conductor and the dielectric outside the conductor cannot be considered in a governing equation, many works put one of the material properties inexact. Meanwhile, the other study calculated electric field and surface charge density by finite element method, but voltage drop in the conductor was assumed to be linear [7]. The method is just approximation, so that the result is inaccurate.

This paper proposes a numerical method to calculate surface charge and electric field in current-carrying conductor. The analysis method based on the finite element method has advantages of wide applicability such as arbitrary shape, nonlinear material and easy formulation. The analysis procedure is two steps: the first step is for inside field of conductor region and the second step is for outside field. The analysis result of the first step is used in the boundary condition of the second step analysis. The surface charge density on the conductor is calculated using the outside electric field on the conductor surface. To prove the feasibility of the proposed method, the down conductors, which the shapes are different, are tested and the geometry effect is examined.

II. CALCULATION OF SURFACE CHARGE AND ELECTRIC FIELD

Since the governing equation in the conductor region differs from the one in the region outside the conductor, the inside and outside fields cannot be analyzed simultaneously. Therefore, the fields should be calculated sequentially. First, the region in the conductor is only analyzed. Dirichlet and homogeneous Neumann conditions are used on the electrodes and the remaining conductor surface, respectively. The governing equation in the conductor is as follows.

$$\nabla \cdot \left(-\sigma \nabla V\right) = 0. \tag{1}$$

where, σ is the electric conductivity and V is the electrical potential. Then, the outside region is analyzed. The solution of (1) on the conductor surface is used as Dirichlet condition. The governing equation outside of the conductor is as follows.

$$\nabla \cdot \left(-\varepsilon \nabla V\right) = 0. \tag{2}$$

where, ε is the electric permittivity. As a result, the electric potential and so the electric field in the whole analysis region is calculated. The distribution of the surface charge on the conductor is calculated by Gauss's law.

$$\rho_s = -\varepsilon \frac{\partial V}{\partial n}. \tag{3}$$

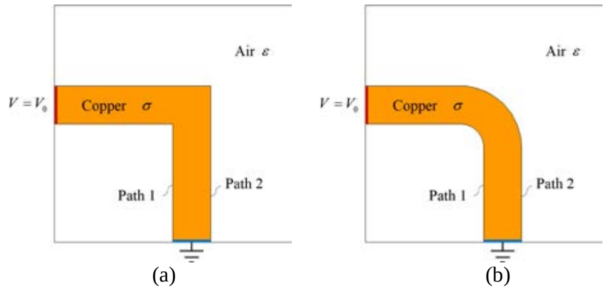
where, ρ_s is the surface charge density and $\mathbf{n}$ is the unit vector in the normal direction to the surface.

III. NUMERICAL EXAMPLE

The down conductor models shown in Fig. 1 are analyzed using the proposed method to examine the bending effect. The red and blue surface indicate two electrodes of each model. The conductivity of the copper is 5.99x10^7 S/m.

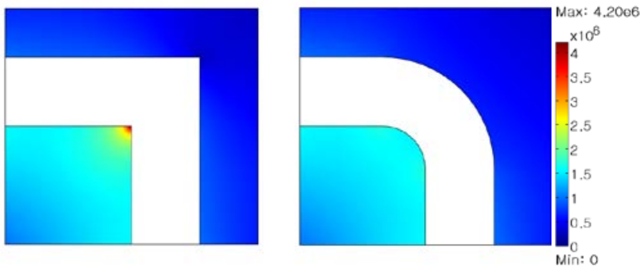
Fig. 2 shows the electric field distributions near the bent part of each model. The field at the inner point of bent part is commonly the strongest and the point is surely close the facility structures. When the field is strong enough to cause shock, the effect can directly damage the facility. However, the field intensity of the sharp bent model is about three times stronger than the smooth bent model which the fillet radius is similar to the diameter of the copper wire. In other words, the quite small rounding of the conductor can considerably alleviate the electric field and relive the possibility of the shock effect.

Fig. 3 shows the distributions of the surface charge density along to the paths indicated in Fig. 1. The surface charge of the sharp bent model is more abruptly changed than the smooth bent model at the bent part. The distributions of surface charge also represent the abrupt change of the current direction as well as the strong field intensity there.



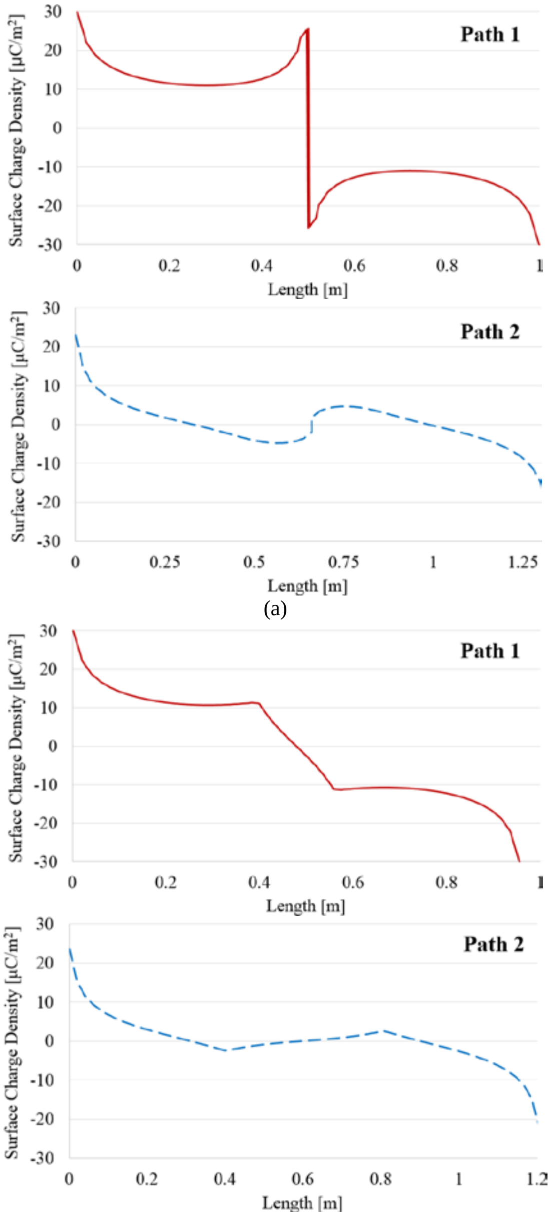
(a) (b)

Fig. 1. Numerical models. (a) Sharp bent model. (b) Smooth bent model.



(a) (b)

Fig. 2. Electric field distributions near bent part. (a) Sharp bent model. (b) Smooth bent model.



(a)

(b)

Fig. 3. Surface charge density distributions along two paths. (a) Sharp bent model. (b) Smooth bent model.

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